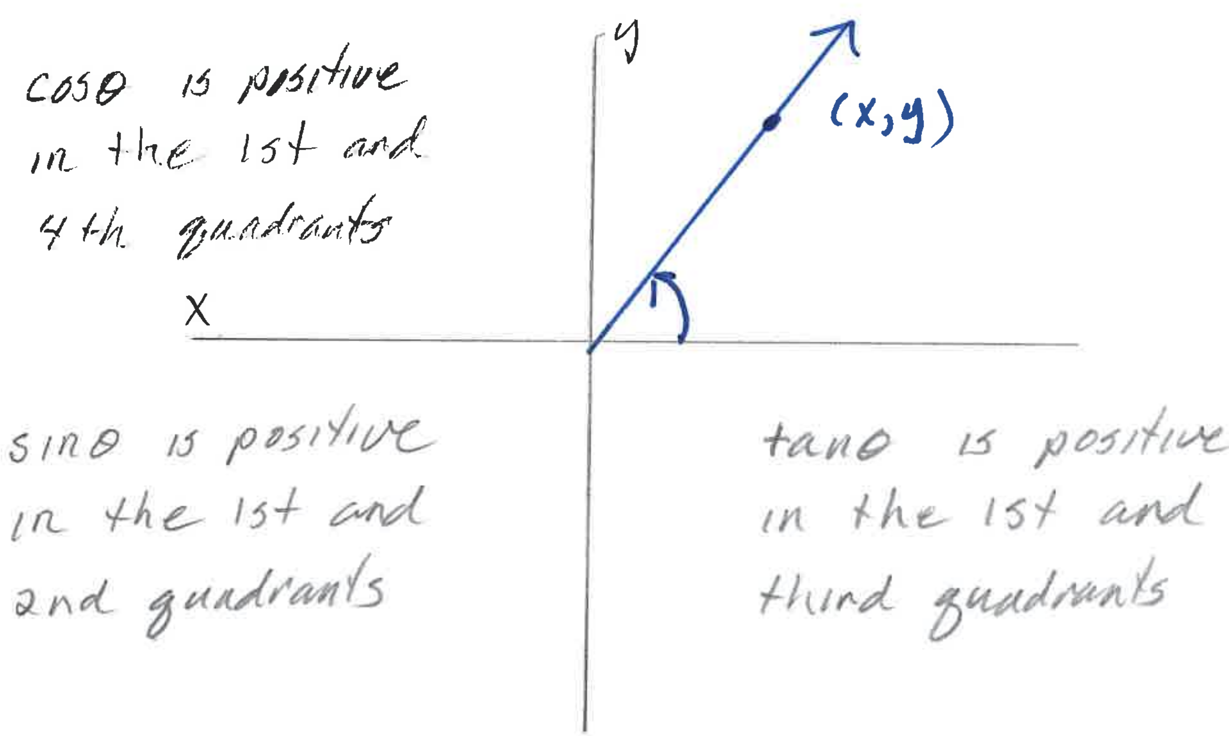
Section 6.3 Trigonometric Functions of Any Angle

Defining the trig functions using the Cartesian Coordinate System:

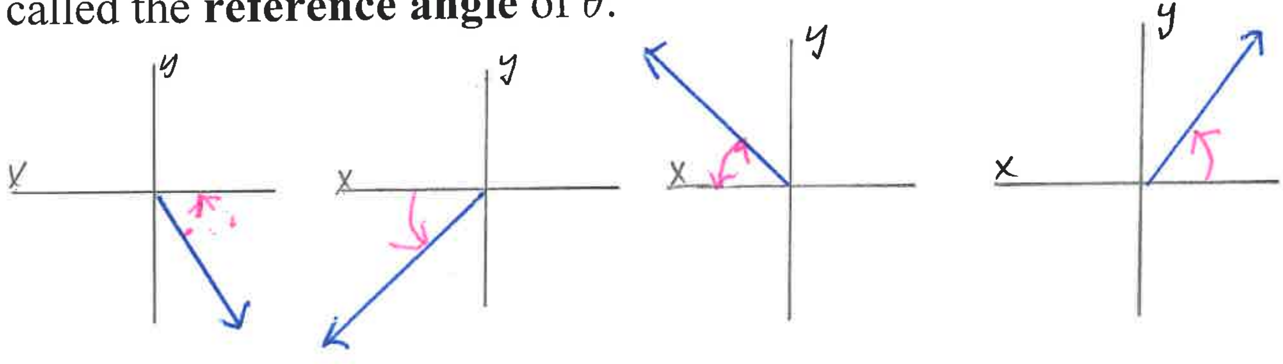
Let θ be an angle in standard position. Let (x, y) be <u>any</u> point P that lies on the terminal side of θ and let r be the distance from the origin to the point P:

$r = \sqrt{x^2 + y^2}$

$sin\,\theta = \frac{y}{r}$	$cos\,\theta = \frac{x}{r}$	$tan\,\theta = \frac{y}{x}$
$csc\,\theta = \frac{r}{y}$	$sec\,\theta = \frac{r}{x}$	$cot\,\theta = \frac{x}{y}$



Definition:
Given an angle θ in standard position. The acute angle that is formed using the terminal side of θ and the x-axis is called the **reference angle** of θ.



To find the value of a trigonometric function of any angle θ you can determine the function value for the reference angle θ_r and then affix the appropriate sign ($-$, $+$) depending on the quadrant in which θ lies.

Think About It:
What angles will have the same sine as the angle whose measure is 80°? What angles will have the same cosine as the angle whose measure is 80°?

To find the value of a trigonometric function of any angle θ you can determine the function value for the reference angle θ_r and then affix the appropriate sign $(-, +)$ depending on the quadrant in which θ lies.

Evaluate the following trigonometric expressions without a calculator:

$sin\ 135^o = \frac{\sqrt{2}}{2}$

reference angle is 45^o

$cos\,(-150^o) = -\frac{\sqrt{3}}{2}$

reference angle is 30^o

$tan\ \frac{13\pi}{6} = \frac{\sqrt{3}}{3}$

reference angle is 30^o or $\frac{\pi}{6}$

$sec\ \frac{7\pi}{4} = \sqrt{2}$

reference angle is $\frac{\pi}{4}$ or 45^o

$cos\ 103\pi = -1$

reference angle is 0^o.

$103\pi = 51(2\pi) + \pi$

$sin\left(-\frac{23\pi}{4}\right) = \frac{\sqrt{2}}{2}$

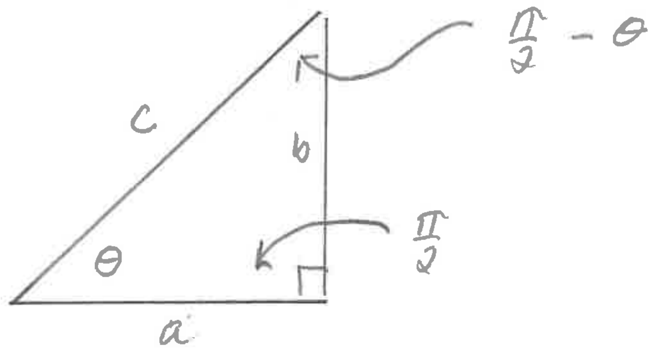
$-\frac{23\pi}{4} = -\frac{16\pi}{4} - \frac{7\pi}{4}$

$= -4\pi - \frac{7\pi}{4}$,

reference angle is $\frac{\pi}{4}$

Complimentary Angles are angles whose sum is a right angle. In other words, the compliment of the angle θ is the angle $\frac{\pi}{2} - \theta$.

Investigate the relationship between the sine and cosine of complimentary angles:



Cofunction Identities:

$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$; $\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$; $\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$, $\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$

Reciprocal Identities:

$\csc\theta = \frac{1}{\sin\theta}$; $\sec\theta = \frac{1}{\cos\theta}$; $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cot\theta = \frac{\cos\theta}{\sin\theta}$